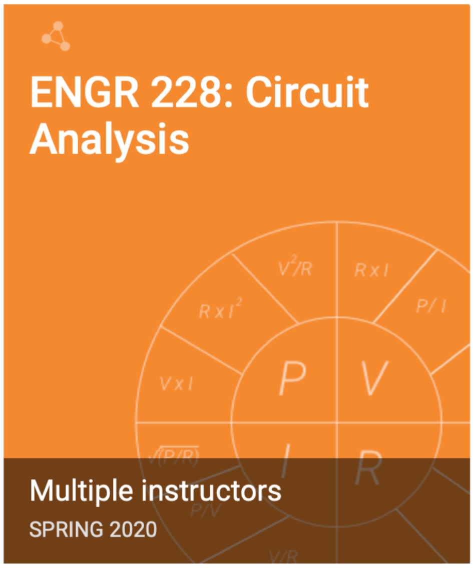


Chapter 5.4

First Order Circuits

Inductance

Engr228 - Circuit Analysis

Spring 2020

Dr Curtis Nelson

Section 5.4 Objectives

- Learn to:
  - Define the electrical properties of an inductor, including its i-υ relationship and energy equation.
  - Combine multiple inductors when connected in series or in parallel.

Inductance

- Faraday’s law states that voltage is induced in an ideal conductor if a changing current passes through it;
- The induced voltage is proportional to the time rate of change (derivative) of the current;
- As a result, energy is stored in the magnetic field surrounding the wire;
- It is customary to use the symbol L for inductance. The SI unit of inductance is the **Henry** (H), named after American scientist and magnetic researcher Joseph Henry.

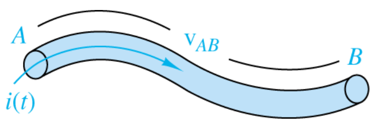
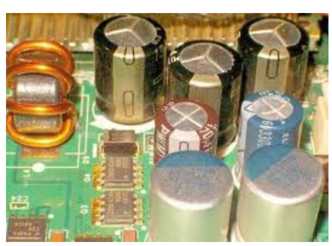


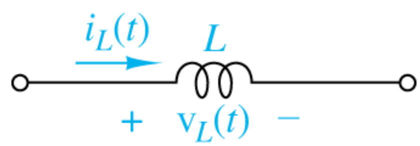
Figure 7.1 Time-varying current flowing through an ideal conductor.

Real Inductors



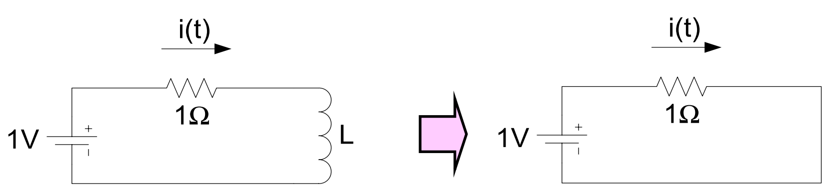
Inductors



- The inductor is often called a coil because physically coiling a wire greatly increases its inductance, especially if it is coiled around a magnetic material.
- The governing voltage and current relationship is:

$$v_L(t) = L\frac{di(t)}{dt}$$

DC Characteristics of Inductors

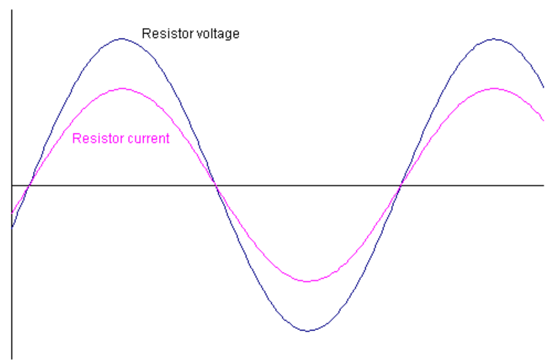


The inductor acts like a "short circuit" at DC because the time rate of current change is equal to zero.

$$v_L(t) = L\frac{di(t)}{dt}$$

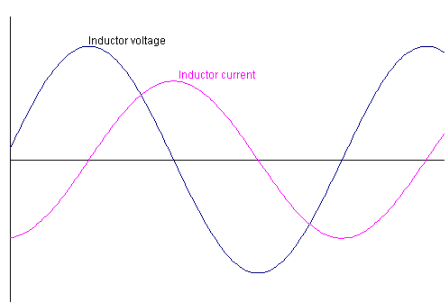
Voltage-Current Relationship in a Resistor

Voltage and current in a resistor are in phase as shown below. The amplitudes may vary due to Ohm's Law, but the phase is the same for the current and the voltage.



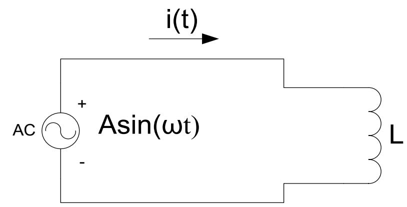
Voltage-Current Relationship in an Inductor

Current and voltage in an inductor are not in phase with each other. **For sinusoidal waves, the voltage across an inductor leads the current through it by 90º.** (In other words, the current lags the voltage by 90º.) In the diagram below, the tall blue waveform represents the voltage across an inductor and the shorter purple waveform represents the current through the inductor.



Inductor VI Characteristics

From the circuit shown at the right, find i(t).



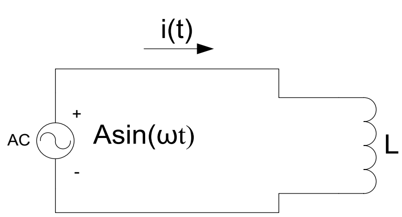
$$v(t) = L\frac{di(t)}{dt}$$

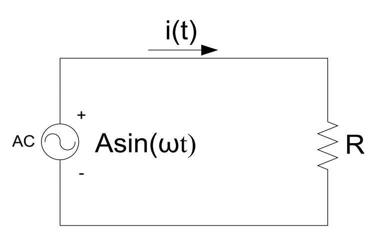
$$i(t) = \frac{1}{L}\int v(t)dt = \frac{1}{L}\int A\sin\omega t dt$$

$$= \frac{A}{L}\int \sin\omega t dt = \frac{A}{L}\left(\frac{-\cos\omega t}{\omega}\right)$$

$$= \frac{A}{\omega L}(-\cos\omega t) = \frac{A}{\omega L}(\sin\omega t - \frac{\pi}{2})$$

Voltage-Current Relationships





$$i(t) = \frac{A}{\omega L}(\sin\omega t - \frac{\pi}{2})$$

Phase shift of -90º

$$i(t) = \frac{A}{R}(\sin\omega t)$$

ωL is known as the ***Inductive Impedance***

Power and Energy in an Inductor

- Instantaneous power is measured in Watts (W):

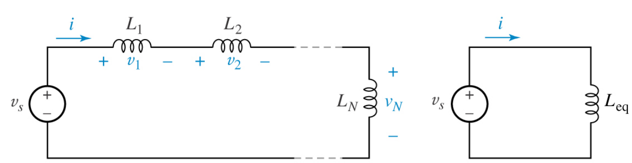
$$p = v_L i_L = L i_L \frac{di_L}{dt}$$

- Energy is the integral of power over a time interval and is measured in Joules (J). Energy is stored in the ***Magnetic*** field surrounding the inductor:

$$w_L(t) = \int_{t_0}^{t} p\,dt$$

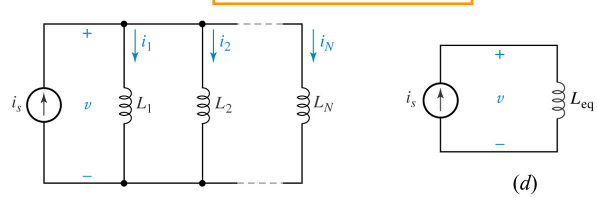
$$w_L(t) = \frac{1}{2} L {i_L}^2$$

Inductor Combinations



(a) (b)

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$



(c) (d)

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

Section 5.4 Summary

- You learned to:
  - Define the electrical properties of an inductor, including its i-υ relationship and energy equation.
  - Combine multiple inductors when connected in series or in parallel.